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CALCULATION AND DESIGN OF CLASS C AMPLIFIERS*

By

FREDERICK EMMONS TERMAN AND WILBER C. ROAKE
(Stanford University, California)

Summary—*A method of calculating the performance of class C triode amplifiers is presented which is based on the assumption that the total space current* (I_p+I_g) *is proportional to* $(E_g+E_p/\mu)^\alpha$, *where* α *is a constant, usually close to three halves. The direct-current and fundamental alternating-current components of such a space current pulse are presented (graphically) as a function of the angle of current flow, for various values of* α, *and it is shown how to obtain the plate current components by estimating the direct grid current, and correcting for the current diverted to the grid.*

It is shown how the results of the analysis may be applied in a straightforward manner to lay out class C amplifiers on paper, and to predict power output, power input, plate loss, etc., for any particular set of operating conditions. The method is applied to several examples and the accuracy shown to be satisfactory for all ordinary design requirements.

IN designing a class C amplifier one normally starts with a particular tube and attempts to realize the optimum operating conditions. This can be done either by setting up the amplifier and following a cut-and-try process, or by making preliminary calculations on paper. The latter method is the most satisfactory since it is much quicker, particularly when large tubes are involved, and is also more certain of yielding the optimum design.

An exact calculation of class C amplifier performance requires that a complete set of characteristic curves be available. With this information one can trace out the plate and grid-current pulses for any operating condition, following the method originally devised by D. C. Prince.[1] This procedure can be simplified somewhat by plotting the tube characteristics in the form of constant current curves as described by Mouromtseff and Kozanowski.[2] Such point-by-point calculations give the exact performance, but unfortunately the amount of labor involved is such that class C amplifiers have usually been built on the cut-and-try basis.

In the last several years a number of approximate methods of de-

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[1] D. C. Prince, "Vacuum tubes as power oscillators," PROC. I.R.E., vol. 11, pp. 275, 405, 527; June, August, and October, (1923).

[2] Mouromtseff and Kozanowski, "Analysis of the operation of vacuum tubes as class C amplifiers," PROC. I.R.E., vol. 23, pp. 752–778; July, (1935).

signing class C amplifiers have been described.[3,4,5,6] These are all based upon the assumption that the plate current of the tube can be represented by some simple law and that the grid current can be neglected. Some of them are also limited to such special operating requirements as a maximum positive grid voltage equal to the minimum instantaneous plate potential. As a result none of these methods of approximate analysis is entirely satisfactory for practical problems.

The purpose of the present paper is to present a means of calculating class C amplifier performance which is both simpler and more accurate than the approximate analyses mentioned, and which at the same time does not neglect the grid current and is not limited to special operating conditions. The method is essentially an extension of a procedure described in a previous paper,[7] and has been in use for several years at Stanford University.

Voltage and Grid Relations in Class C Amplifiers

The circuit and fundamental voltage and current relations of a class C amplifier are illustrated in Fig. 1. The voltage actually applied to the grid of the tube consists of the grid bias E_c plus the exciting voltage E_s. The relations are normally such that at the crest of the cycle the grid is driven appreciably positive and consequently draws some grid current. The voltage actually appearing at the plate of the tube consists of the battery voltage E_b minus the voltage drop E_L in the plate load impedance, and so has the wave shape shown in Fig. 1(a). The phase relations are such that the minimum instantaneous plate potential E_{min} occurs the same part of the cycle as the maximum grid potential E_{max}. The alternating components of the plate and grid voltage are also always sinusoidal since they are developed across sharply resonant circuits.

The plate and grid currents that flow at any instant are the result of the combined action of the plate and grid potentials at that instant, and can be determined from these potentials with the aid of a set of complete characteristic curves of the tube. The plate current is in the form of an impulse flowing for something less than half a cycle. The grid cur-

[3] L. B. Hallman, Jr., "Fourier analysis of radio-frequency power amplifier wave forms," Proc. I.R.E., vol. 20, pp. 1640–1659; October, (1932).

[4] W. L. Everitt, "Optimum operating condition for class C amplifiers," Proc. I.R.E., vol. 22, pp. 152–176; February, (1934).

[5] Burton F. Miller, "Analysis of class B and class C amplifiers," Proc. I.R.E., vol. 23, pp. 496–510; May, (1935).

[6] A. P. T. Sah, "The performance characteristics of linear triode amplifiers," *Science Reports of National Tsinghua University*, Peiping, China, vol. 2, pp. 49 and 83; April and July, (1933).

[7] F. E. Terman and J. H. Ferns, "The calculation of class C amplifier and harmonic generator performance of screen-grid and similar tubes," Proc. I.R.E., vol. 22, pp. 359–373; March, (1934).

rent flows only when the grid is positive, and is usually sharply peaked. In some cases the grid current may reverse and be negative for a portion of the time as a result of secondary emission. The sum (I_p+I_g) of plate and grid currents represents the total space current flowing away from the filament, and is shown in Fig. 1(c). This current always has its peak at the instant when the grid and plate potentials are $E_{\max}$ and $E_{\min}$, respectively.

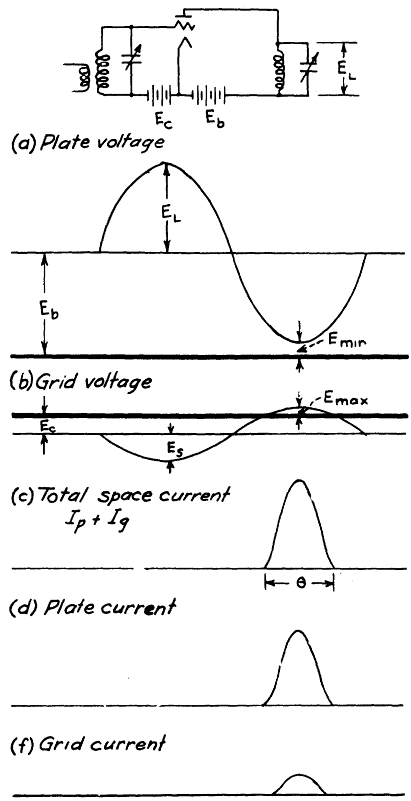

Fig. 1—Circuit and voltage and current relations of class C amplifier.

The average value of the plate current pulse over a complete cycle represents the direct current which will be observed in the plate circuit, while the average value of the grid current pulse is likewise the direct grid current. The power input which must be supplied by the plate battery is the direct plate current times the plate supply voltage E_b. The power delivered to the load is equal to half the product of alternating voltage $E_L=E_b-E_{\min}$ across the load and the crest value of the fundamental frequency component of the plate current impulse.

Basis of Analysis

The analysis given in this paper is based upon the assumption that the total space current (I_p+I_g) can be expressed by the following mathematical relation

$$\text{total space current} = (I_p + I_g) = K(E_g + E_p/\mu)^\alpha \tag{1}$$

where E_g and E_p are instantaneous plate and grid voltages, respectively, μ is the amplification factor of the tube, and K and α are constants. The exponent α in (1) is normally very close to three halves and would be exactly three halves if the tube were perfectly symmetrical and had full space-charge saturation. Actually it is found that the characteristic of ordinary power tubes follows the relationship given in (1) reasonably well.

Upon the assumption that the space current is of the form given by (1), one can plot curves which give the relationship between the direct current and fundamental frequency components of the total space current in terms of the maximum space current I_m and the number of electrical degrees θ during which the plate current flows. Such curves have been calculated according to the method given in the Appendix for various values of α between 1.0 and 2.0, and are presented in Fig. 2. The importance of these curves is that they enable one to determine the direct-current and fundamental alternating-current components of the space current impulse without the necessity of resorting to point-by-point calculations.

Design Procedure

The procedure for designing a class C amplifier making use of the curves of Fig. 2 involves a sequence of steps as outlined below.

First. The first step is the choice of a suitable value of crest space current I_m. The maximum value permissible is determined by the electron emission which the filament is capable of producing, and in order to use the full possibilities of the tube it is usually desirable to select the highest possible value of I_m. With tungsten filaments it is common practice to make I_m equal substantially the full emission from the filament in the case of class C amplifiers, and perhaps two thirds this for modulated and class B amplifiers where linearity is important. With thoriated tungsten filaments the deterioration during life is such that factors of safety of three to seven are common, with the exact value depending upon how thoroughly the tube has been evacuated. The characteristics of oxide-coated filaments vary so much that still higher factors of safety must be employed with them.

Second. After an appropriate value of maximum space current has been determined one next selects a combination of values for maximum grid potential E_{max}, and minimum plate potential E_{min}, that will draw this total space current. What is desired is the lowest possible value of minimum plate voltage E_{min} because the lower the plate voltage the higher will be the efficiency of the amplifier. In order to draw the full space current with a low minimum space current it is then necessary to make the maximum grid potential E_{max} large. However, the maximum grid potential should never under any conditions exceed the minimum plate voltage since this will cause the grid current to be excessive.

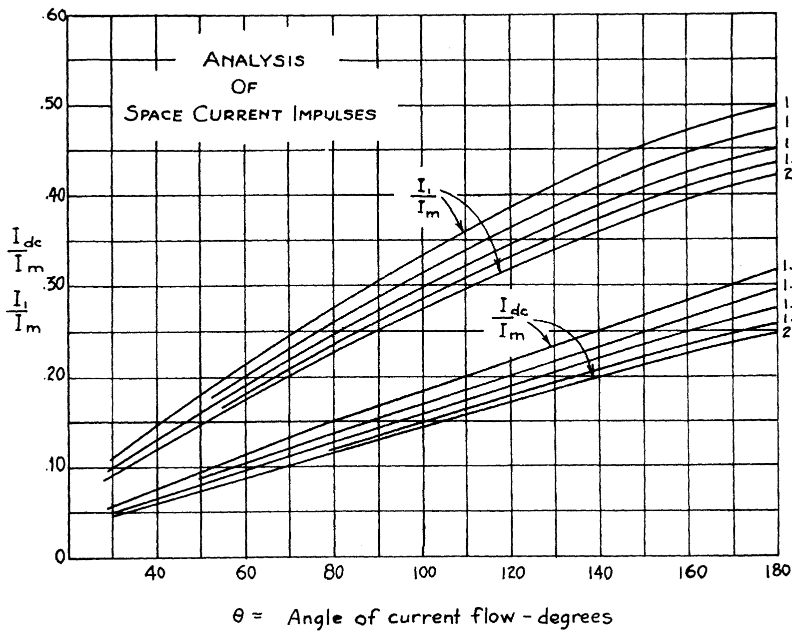

Fig. 2—Curves giving the direct-current and fundamental frequency components of the space current impulse as a function of the angle of flow, for various values of α in equation (1).

This large grid current is diverted away from the plate, thereby reducing the output power, and also represents large driving power since it means a high power consumption in the grid circuit. The usual practice is to make the maximum grid potential approximately equal to the minimum plate potential with tubes which operate at a few thousand volts plate potential, while with large water-cooled tubes the maximum grid potential is normally considerably less, perhaps one half to one fifth of the minimum plate potential.

When complete tube characteristics are available one can determine from them combinations of E_{min} and E_{max} which will draw the desired value of total space current. When complete characteristics are not available, satisfactory results can be obtained by plotting the total space current (I_p+I_g) as a function of effective anode voltage (E_g+E_p/μ) on log-log paper, covering the range of values which can be taken in the usual point-by-point way without overheating the tube. This curve will be substantially a straight line and can then be extrapolated to the desired total space current I_m.

Third. One is now ready to select the fraction of the cycle during which plate current flows, and to calculate grid bias and exciting voltage. The fraction of the cycle is determined as a compromise between a number of conflicting factors, since a small value gives high plate efficiency but results in small power output and large driving power, while a large value gives a large output, but makes the plate efficiency low. The factors normally balance when the current flows for something between 90 and 180 electrical degrees.

The grid bias E_c required to cause the plate current to flow for θ electrical degrees is given by the equation[8]

$$\text{grid bias} = E_c = \frac{E_b}{\mu} + \left(E_{\max} + \frac{E_{\min}}{\mu}\right)\frac{\cos\theta/2}{1-\cos\theta/2} \tag{2}$$

where $E_{\max}$, $E_{\min}$, E_b, and μ have the same definitions as above. With the grid bias E_c and the maximum positive grid potential $E_{\max}$ both known, the exciting voltage is $(E_{\max}+E_c)$.

Fourth. One is now ready to calculate the power relations. The first step is the determination of the components of the total space current for the I_m and the angle of flow selected above. This is done with the aid of the curves in Fig. 2 using a value of the exponent α which experience indicates is desirable, and which in the absence of information to the contrary can be assumed to be three halves.

The total space current determined in this way is divided between the plate and grid electrodes of the tube. In order to determine the effect of the grid current it is necessary to make an estimate based on experience, of the fraction of the total direct space current that will be diverted to the grid. This percentage is commonly ten to twenty-five per cent in air-cooled tubes operating at plate potentials up to several thousand volts, while with water-cooled tubes it is less, with the grid

[8] This equation follows from the fact that at the instant the plate current stops flowing the signal and load voltages are $\theta/2$ degrees from their crest values, so that at this instant the effective anode voltage is

$$\left(\frac{E_b - (E_b - E_{\min})\cos\theta/2}{\mu} - E_c + (E_{\max} + E_c)\cos\theta/2\right),$$

and this must equal zero.

current often reversing as a result of secondary electron emission. After the probable grid current has been estimated, the direct plate current is obtained by subtracting the direct grid current from the direct-current component of the total space current I_m. In the event that the grid current is negative, the direct component of the plate current will be larger than the direct-current component of the total space current.

The fundamental alternating-current component of the total space current is likewise divided between grid and plate electrodes, with the amount going to the grid very nearly equal to twice the direct current component of the grid current. This comes about because most of the grid current flows during the very crest of the cycle, and as shown in a previous paper[7] this is equivalent to an alternating component that is twice the direct-current value of the grid current. The alternating component of the plate current is hence the alternating-current component of the total space current as obtained from Fig. 2 minus twice the direct grid current. The power input to the class C amplifier is now the product of battery voltage and direct plate current, or

$$\text{power input} = E_b \times I_{dc} \tag{3}$$

where I_{dc} is the direct plate current. Likewise the power delivered to the load is equal to half the product of alternating plate current and alternating voltage developed across the load, or

$$\text{power output} = \frac{(E_b - E_{\min})I_{ac}}{2} \tag{4}$$

where I_{ac} is the crest value of the fundamental frequency component of the plate current. The plate dissipation is the difference between these two powers and the efficiency is their ratio.

The grid driving power is then approximately equal to the direct grid current as estimated above times the crest value of the exciting voltage.[9]

Fifth. The above steps give a complete solution of the class C amplifier for the assumed values of I_m, θ, $E_{\max}$, and $E_{\min}$. If the results obtained are not satisfactory, one can make a new choice of initial operating conditions and recalculate to obtain more nearly the optimum performance.

[9] This is because to a first approximation the grid current can be assumed to flow when the grid exciting voltage is at its crest. *See* F. E. Terman, "Radio Engineering," p. 234. Experimental work indicates that power calculated in this way is in the order of five to ten per cent high. *See* H. P. Thomas, "Determination of grid driving power in radio-frequency power amplifiers," Proc. I.R.E., vol. 21, pp. 1134–1141; August, (1933).

Example

In order to show in detail how the above procedure is carried out, a class C amplifier will now be designed using a type 800 tube at 1000 volts plate potential. The complete characteristic curves of such a tube as given by the manufacturer are shown in Fig. 3.

The peak emission I_m will be taken as 407 milliamperes. This is arrived at by assuming that initially the thoriated tungsten filament is capable of emitting 100 milliamperes per watt of heating power, and then allowing a factor of safety of six to provide for deterioration during life. With a small tube such as the 800, $E_{\min}$ and $E_{\max}$ will normally be about equal. Assuming this equality and referring to Fig. 3, it is found that 123 volts on both grid and plate will draw a space current of 407 miliamperes. The next step is the selection of the angle of current flow. A value of 120 degrees represents a reasonable compromise between high efficiency and large output and will be tentatively selected. From the curves of Fig. 2, on the assumption that $\alpha = 3/2$, the factors for direct plate current and alternating plate current are 0.19 and 0.35, respectively, so that the corresponding components of the total space current are $407 \times 0.19 = 77.5$ milliamperes direct current, and $407 \times 0.35 = 142$ crest milliamperes of fundamental frequency. It is now necessary to make allowance for the part of the total space cur-

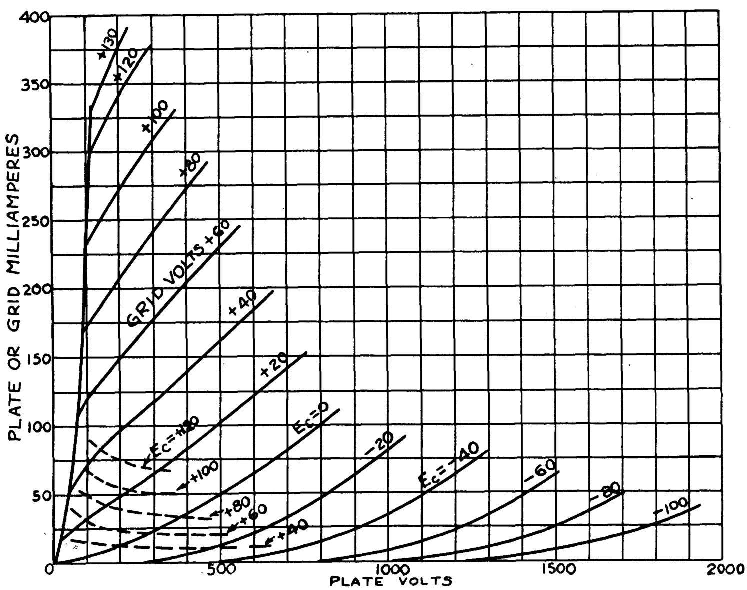

Fig. 3—Characteristic curves of type 800 tube.

rent diverted to the grid. Assuming that twenty per cent of the total direct space current will be diverted to the grid as reasonable for small tubes, the direct grid current will be 15.5 milliamperes. The direct plate current is then $77.5-15.2=62$ milliamperes, and the fundamental frequency component of the plate current is similarly $142-2\times 15.5=111$ milliamperes crest value.

The power input to the plate circuit is the product of direct plate current and plate voltage, or $1000\times 0.062=62$ watts, while the power output is half the product of crest alternating plate current and crest alternating voltage across the load, and so is $0.111\ (1000-123)/2=49$ watts. The plate loss is $62-49=13$ watts, and the efficiency is $49/62=79$ per cent.

The grid bias required as calculated by (2) is found to be 198 volts. The crest alternating driving voltage is (E_c+E_{max}) or 321 volts, and the grid driving power is to a first approximation $321\times 0.0155=5.0$ watts. The load impedance that is required is the ratio of alternating voltage (E_b-E_{min}) to the alternating-current component of the plate current, and so is $(1000-123)/0.111=7900$ ohms.

If the above results do not represent the desired operating conditions, one can readily make a new set of calculations on the basis of a new value of angle of current flow, or a different combination of E_{min} and E_{max}, or both. In particular it will be noted that although the above operating conditions develop the normal rated output, the plate losses are considerably lower than the maximum allowable loss, and that increased output at a somewhat lower efficiency could be obtained by using a larger angle of current flow. Also, by making $E_{min}>E_{max}$, one could reduce the grid current, and hence the driving power.

Accuracy of Calculations

The only approximations involved in the method of analyzing class C amplifiers outlined above are the uncertainty regarding the exact amount of grid current, and the assumption that the exponent α in (1) is constant. The necessity of making a guess as to the grid current need not introduce appreciable error since the grid current is always a small proportion of the total space current, and therefore a considerable percentage error in grid current represents only a very small percentage error in the plate current components. Also any error in estimating grid current alters the calculated power input, power output, and tube losses, all in substantially the same proportions and therefore has little effect on the plate efficiency.

The assumption that the exponent α in (1) is constant is found to be substantially correct over the essential part of the tube characteristics provided the peak space current I_m is not so great as to approach saturation by insufficient electron emission. In the case of tungsten filament tubes operated with total space currents very close to the peak emission available, some saturation effects are normally found. Even then the error that results is not particularly great, and alters both power input and power output in about the same proportion, so that the predicted plate efficiency will still be almost exactly correct.

With a little experience it is possible to make fairly accurate allowances for these incipient saturation effects. Thus when the value of I_m is taken from complete characteristic curves any tendency toward saturation that is present will cause the calculated input and output powers to both be slightly low. On the other hand, when one extrapolates a curve of I_m as a function of $(E_g+E_{p/\mu})$ to get the operating conditions required to draw the maximum space current I_m, the calculated input and output powers will both be slightly higher than the true values.

The accuracy obtainable with analyses based upon the curves of Fig. 2 is shown by the following examples.

Example One. A point-by-point calculation of the performance of the class C amplifier considered above, using the same $E_{\min}$, $E_{\max}$, and angle of current flow, gives the following results:

	By analysis using Fig. 2	By exact point-by-point calculation
Direct plate current	62. ma	63.9 ma
Direct grid current	15.5 ma	12.6 ma
Alternating plate current	111. ma	116. ma
Power input	62 watts	64 watts
Power output	49 watts	51 watts
Plate loss	13 watts	13 watts
Plate efficiency	79 per cent	79.5 per cent

It will be noted that if the grid current had been more accurately estimated the agreement would have been practically perfect.

Example Two. Mouromtseff and Kozanowski[2] have given on page 761 of their recent paper the results obtained from a point-by-point analysis of a tube having characteristics such as shown in their Fig. 1, under the following conditions:

$$E_b = 20{,}000 \text{ volts}$$
$$E_{\min} = 6{,}500 \text{ volts}$$
$$E_{\max} = 740 \text{ volts}$$
$$\text{Grid bias} = 1{,}400 \text{ volts}$$
$$\mu = 20$$
$$\text{Direct grid current} = -100 \text{ milliamperes}$$

From these data, the angle of current flow is found by (2) to be 148 degrees. Reference to the characteristic curves given for the tube shows that the maximum space current is 7.2 amperes. With these data, the results calculated with the aid of the curves of Fig. 2 for $\alpha = 3/2$ are given in the following tabulation, together with the exact results calculated by Mouromtseff and Kozanowski:

	By Fig. 2	Exact	Percentage Difference
Input power	35.5 kw	37.1 kw	−4.3 per cent
Output power	21.0 kw	21.75 kw	−3.4 per cent
Plate loss	14.5 kw	15.35 kw	−5.5 per cent
Efficiency	59.1 per cent	58.8 per cent	+0.5 per cent

It is apparent from the above examples that the degree of accuracy obtainable is quite satisfactory for preliminary calculations and can be safely used as the basis for circuit design.

Practical Adjustment of Class C Amplifiers to Realize the Desired Conditions

After the desired conditions have been calculated, one still must realize these in actual operation. The first step is to obtain the appropriate grid bias and plate supply potentials, after which the load is coupled into the plate circuit a reasonable amount. If a positive peak voltmeter is available the excitation is adjusted to give the desired positive grid potential $E_{\max}$, after which the load coupling is varied until a point is found where the direct plate current approximates the calculated value, and further reduction in coupling causes the grid current to increase rapidly and become excessive. If a positive peak voltmeter is not employed, the proper procedure is to adjust the grid excitation until the total space current $I_p + I_g$ approximates the desired value, after which the load coupling is adjusted as above. It may then be necessary to readjust the excitation and load coupling slightly to realize the desired total space current and direct plate current.

The amount the grid is driven positive can be measured directly by means of a peak vacuum tube voltmeter such as illustrated in Fig. 4. This instrument has been described elsewhere[7] so will be considered here only briefly. It makes use of a diode tube, preferably an 879 or similar tube capable of standing a high voltage and having relatively low interelectrode capacity. The anode is connected directly to the grid of the class C amplifier while the cathode is biased positive with respect to the amplifier cathode by the potentiometer P until the milliammeter M just begins to show current. Under this condition the cathode potential is substantially equal to the most positive potential reached by the rectifier anode, so that voltmeter V then reads E_{max} directly.

If desired it is also possible to arrange a modified peak voltmeter (trough meter) which is capable of reading the minimum plate potential E_{min}, but this is not really required. Voltmeters for this purpose are described elsewhere.[10]

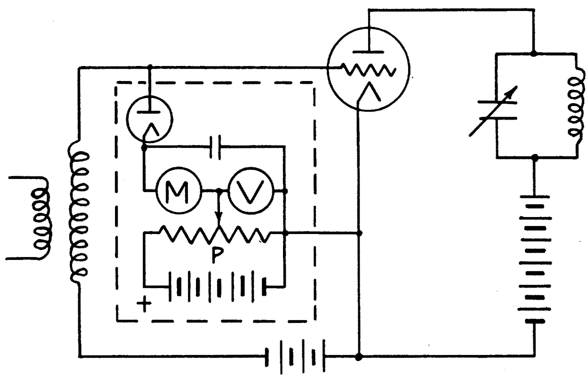

Fig. 4—Peak voltmeter for adjusting E_{max} to desired value.

Appendix

The curves shown in Fig. 2 can be calculated by the following procedure:

The waves to be analyzed have a shape corresponding to (1). When the exponent α is unity this is essentially a section of a sine wave having a duration θ and a height above the base line of $I = I_m$, as shown in Fig. 5. The equation of such a wave is

$$i = I\frac{(\cos\beta - \cos\theta/2)}{1 - \cos(\theta/2)} \quad \text{for } \beta < \theta/2 \qquad (5)$$

$$i = 0 \qquad \text{for } \beta > \theta/2$$

where i is the amplitude above the axis at β degrees from the crest. If the exponent α in (2) is not unity, then the equation of current is simply the right-hand side of (5) raised to the α power, with I_m substituted for I^α. That is,

$$i = I_m\left[\frac{\cos\beta - \cos\theta/2}{1 - \cos\theta/2}\right]^\alpha \quad \text{for } \beta < \theta/2 \qquad (6)$$

$$i = 0 \qquad \text{for } \beta > \theta/2.$$

[10] F. E. Terman, "Measurements in Radio Engineering," McGraw-Hill Book Co.

The value I_{dc}/I_m is then the right-hand side of (6) averaged over a cycle, and divided by I_m, or

$$\frac{I_{dc}}{I_m} = \frac{1}{\pi}\int_0^{\beta=\theta/2}\left[\frac{\cos\beta - \cos\theta/2}{1 - \cos\theta/2}\right]^\alpha d\beta. \tag{7}$$

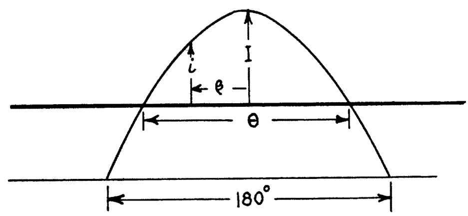

Fig. 5—Figure illustrating notation in equation (5).

The fundamental frequency component of the wave of (6) is found in the usual Fourier method by multiplying by cos β and averaging over the cycle. This yields

$$\frac{I_1}{I_m} = \frac{2}{\pi}\int_0^{\beta=\theta/2}\left[\frac{\cos\beta - \cos\theta/2}{1 - \cos\theta/2}\right]^\alpha \cos\beta\, d\beta. \tag{8}$$

The integrations involved in (7) and (8) cannot be carried out in a simple mathematical manner for values of α that are not integers. The authors therefore used point-by-point methods in deriving Fig. 2 and after trying out several procedures believe that Simpson's rule is the most satisfactory. According to this, the base θ of the curve is divided into n equal parts, where n is an even number. The area under the curve is then

$$\text{area} = \tfrac{1}{3}h[y_0 + 4(y_1 + y_3 + y_5 \cdots y_n - 1) + 2(y_2 + y_4 + y_6 \cdots y_n - 2) + y_n] \tag{9}$$

where h is the distance between adjacent ordinates ($h = \theta/n$), and y_0, y_1, y_2, etc., are the heights of the curve for the various positions along the base of the curve. The accuracy increases as the number of intervals n increases. Enough points were used in obtaining Fig. 2 to insure an error of less than one per cent.