

Q-based design method for T network impedance matching

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Abstract

Impedance matching using passive network is very important in the design of RF and microwave circuits to achieve maximum power transfer, minimum reflection, and adequate harmonic rejection. This paper investigates the T network impedance matching analytically. A practical guide is provided for systematic network design based on the desired harmonic rejection specification. Expressions of the frequency response and harmonic rejection in term of the loaded Q are established. The limits of the network are analytically demonstrated. It is shown that the third harmonic rejection is about 12 dB higher than second harmonic rejection provided that the loaded Q is larger than twice the minimum Q.

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1. Introduction

Maximum power transfer from source to load will occur when the load impedance is equal to the complex conjugate of the source impedance. As the wave power is totally absorbed by the load, reflection is minimized and transmission of constant amplitude over the frequency band of interest can be maintained. Impedance transformation using passive network is usually performed to achieve the desired impedance matching condition for optimal power flow. Tuning of matching network is commonly employed in RF/microwave active circuit and antenna design. With the ever-increasing speed of ICs and computer systems, digital signal must be treated much like an analog signal in order to ensure signal integrity. Transmission line theory must be applied in the design of printed-circuit board traces. Impedance matching becomes an important concern in the design of high-speed electronic circuits to prevent electromagnetic interference.

The most frequently used matching networks are the L network, Pi network and T network. Considerations in selecting a matching network may include the simplicity, bandwidth, manufacturing feasibility, and ease of tuning

[1–10]. When L network is used, a unique solution may be found for a given impedance matching problem. The matching bandwidth and harmonic rejection are determined by the load impedance. In order to increase the quality factor Q, therefore increases the harmonic rejection (at the expense of narrower matching bandwidth), additional reactive component may be added to form a Pi or T network. However, with three component variables and two simultaneous equations to solve, virtually unlimited combinations of inductor and capacitor values may be used to achieve the desired impedance matching condition. One may select an inductor/capacitor combination and end up getting a Q that is larger or smaller than the desired value. An optimization process will be required to achieve the design goal.

Low-pass filter solution is usually preferred, attributed to its advantages in harmonic rejection. A Pi network may be formed with a series inductors and two shunt capacitors as shown in Fig. 1. A T network may be formed with two series inductors and a shunt capacitor as shown in Fig. 2. Because inductor is generally more expensive than capacitor, T network is more costly to implement and therefore Pi network is preferred by many RF designers. The Pi network design method is well established over the years. In the production of microwave integrated circuits (MIC) and monolithic microwave integrated circuits (MMIC),

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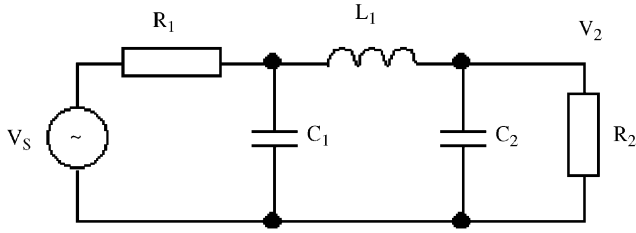


Fig. 1. Pi network impedance matching.

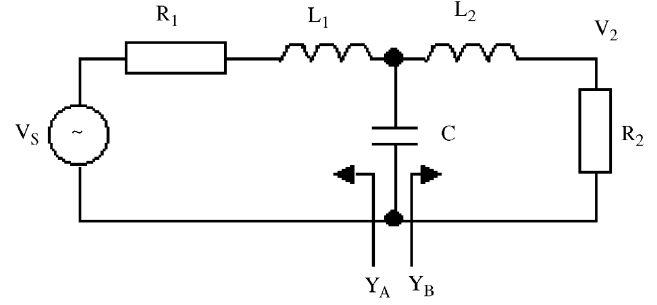


Fig. 2. T network impedance matching.

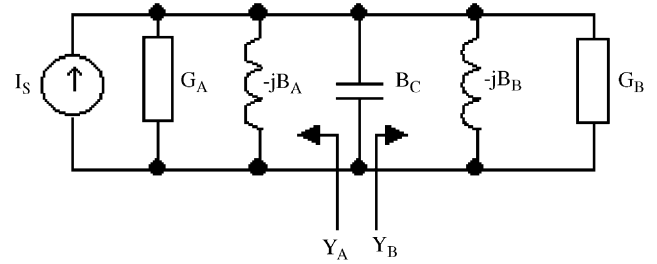


Fig. 3. Equivalent circuit of the T network.

lump inductors and capacitors can be fabricated on the substrate material. There is no cost difference in implementing Pi or T network. Hence, a Q-based T network design method is presented in this paper to complement the Pi network design method.

Single-frequency matching techniques play a very important role in broadband impedance matching [11–17]. Matching network may be designed for a midband frequency to provide an initial solution of broadband matching problems in solid-state microwave amplifiers.

2. Design of T network impedance matching

Consider the T network in Fig. 2. The goal of impedance matching is to obtain maximum power transfer and at the same time provide the desired harmonic rejection. The T network can be represented by the equivalent circuit in Fig. 3.

$$Y_A = G_A - jB_A, \quad (1a)$$

$$Y_B = G_B - jB_B. \quad (1b)$$

We can define

$$Q_1 = \frac{X_{L1}}{R_1} = \frac{B_A}{G_A}, \quad (2a)$$

$$Q_2 = \frac{X_{L2}}{R_2} = \frac{B_B}{G_B}. \quad (2b)$$

The conjugate matching will result in

$$G_A = G_B \quad (3a)$$

and

$$B_C = B_A + B_B. \quad (3b)$$

The loaded quality factor Q_0 of the network can be defined as

$$Q_0 = \frac{B_A + B_B}{G_A + G_B} = \frac{Q_1 + Q_2}{2}. \quad (4)$$

Using a series-to-parallel circuit conversion formula,

$$\frac{1}{G_A} = R_1(1 + Q_1^2), \quad (5a)$$

$$\frac{1}{G_B} = R_2(1 + Q_2^2), \quad (5b)$$

$$\frac{1}{B_A} = Q_1 R_1 \left(1 + \frac{1}{Q_1^2} \right), \quad (5c)$$

$$\frac{1}{B_B} = Q_2 R_2 \left(1 + \frac{1}{Q_2^2} \right). \quad (5d)$$

We can combine Eqs. (5a) and (5b) based on the conjugate matching condition of Eq. (3a), and combine Eqs. (5c) and (5d) based on Eq. (3b),

$$\frac{1 + Q_1^2}{1 + Q_2^2} = \frac{R_2}{R_1}, \quad (6a)$$

$$B_C = \frac{2Q_0}{R_1(1 + Q_1^2)} = \frac{2Q_0}{R_2(1 + Q_2^2)}. \quad (6b)$$

Eq. (6a) can be rewritten as

$$Q_1^2 = \frac{R_2}{R_1} (1 + Q_2^2) - 1. \quad (7)$$

In order for $Q_1 > 0$ and $Q_2 > 0$,

$$Q_2 > \sqrt{\frac{R_1}{R_2} - 1} \quad \text{when } R_1 > R_2, \quad (8a)$$

$$Q_1 > \sqrt{\frac{R_2}{R_1} - 1} \quad \text{when } R_1 < R_2. \quad (8b)$$

From Eqs. (4) and (8a), we obtain

$$Q_0 > \frac{1}{2} \sqrt{\frac{R_1}{R_2} - 1} \quad \text{when } R_1 > R_2. \quad (9a)$$

And from Eqs. (4) and (8b), we obtain

$$Q_0 > \frac{1}{2} \sqrt{\frac{R_2}{R_1}} - 1 \quad \text{when } R_1 < R_2. \quad (9b)$$

These limits represent the minimum Q_0 achievable with a T network impedance matching. They also represent the conditions whereby one of the inductances is zero and the circuit reduces to an L network. In other words, the loaded Q of a T network will always be larger than the loaded Q of an L network. In the design of T network impedance matching, one must not select $Q_0 < Q_{0(\min)}$.

We can define $k = R_1/R_2$. Solving Eqs. (4) and (7), and selecting the solutions where both Q_1 and Q_2 are larger than zero, yields

$$Q_1 = \frac{2Q_0 - \sqrt{4kQ_0^2 - (k-1)^2}}{1-k}, \quad (10a)$$

$$Q_2 = \frac{2kQ_0 - \sqrt{4kQ_0^2 - (k-1)^2}}{k-1}. \quad (10b)$$

The value of k can be either larger or smaller than 1. Once the values of Q_1 and Q_2 are calculated based on the desired Q_0 , the desired inductances L_1 and L_2 , and capacitance C , can be calculated from Eqs. (2) and (6b).

$$L_1 = \frac{R_1 Q_1}{\omega_m}, \quad (11a)$$

$$L_2 = \frac{R_2 Q_2}{\omega_m}, \quad (11b)$$

$$C = \frac{2Q_0}{\omega_m R_1 (1 + Q_1^2)} = \frac{2Q_0}{\omega_m R_2 (1 + Q_2^2)}, \quad (11c)$$

where ω_m is the angular frequency of the impedance matching network.

3. Guideline for selection of Q_0

Refer to the T network in Fig. 2. Defining the voltage transfer function $H(s) = V_2/V_s$, and using a basic circuit

analysis method, we obtain

$$H(s) = \frac{R_2}{s^3 C L_1 L_2 + s^2 C (R_1 L_2 + R_2 L_1) + s (C R_1 R_2 + L_1 + L_2) + (R_1 + R_2)}, \quad (12)$$

where $s = j\omega$. The power transfer function can be defined as

$$\frac{P_2(\omega)}{P_A} = \frac{V_2^2/R_2}{V_s^2/2R_1} = 2k|H(\omega)|^2, \quad (13)$$

where $P_A = P_S(\omega_m)$ is the available power under conjugate match condition. At ω_m , the power delivered to the load resistance R_2 is half of P_A , therefore

$$\frac{P(\omega_m)}{P_A} = \frac{1}{2}. \quad (14)$$

By defining selectivity of the network as $S(\omega) = P_2(\omega)/P_2(\omega_m)$, Eqs. (15) can be derived from Eqs. (12)–(14).

$$S(\omega, Q_0, k) = \frac{16k \left[Q_0(k+1) - \sqrt{4kQ_0^2 - (k-1)^2} \right]^2}{\left\{ 2Q_0(k+1)^2 - 2Q_0(k-1)^2(\omega/\omega_m)^2 - 2(k+1)\sqrt{4kQ_0^2 - (k-1)^2} \right\}^2 + \left\{ \left[3(k-1)^2 - 8kQ_0^2 + 2Q_0(k+1)\sqrt{4kQ_0^2 - (k-1)^2} \right] (\omega/\omega_m) + \left[8kQ_0^2 - (k-1)^2 - 2Q_0(k+1)\sqrt{4kQ_0^2 - (k-1)^2} \right] (\omega/\omega_m)^3 \right\}^2} \quad (15)$$

The frequency response of the T impedance matching network is shown in Fig. 4 (for $k = 0.2$ or $Q_{0(\min)} = 1$). The second and third harmonic rejections can be calculated by substituting ω with $2\omega_m$ and $3\omega_m$, respectively. Fig. 5 shows a plot of second and third harmonic rejection performances with respect to the loaded-Q of the network. It can be shown that, for $Q_0 > 2Q_{0(\min)}$, the value of k will have negligible effect on second harmonic rejections

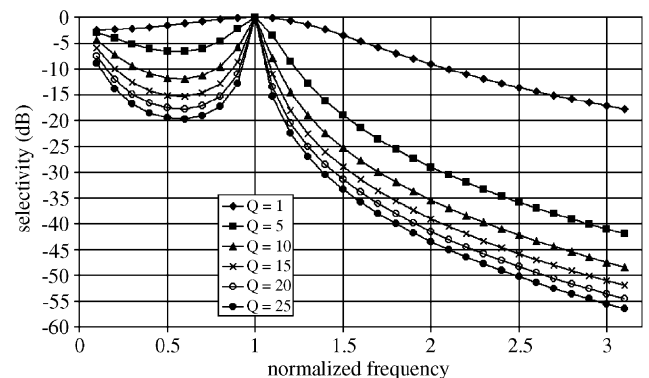


Fig. 4. Frequency response of T impedance matching network.

(within 1 dB error). We can further show that

$$S(2\omega_m, Q_0, k) \approx S(2\omega_m, Q_0, 1) \approx \frac{1}{36Q_0^2}, \quad (16a)$$

$$S(3\omega_m, Q_0, k) \approx S(3\omega_m, Q_0, 1) \approx \frac{1}{576Q_0^2}. \quad (16b)$$

The harmonic rejections in dB is then given by

$$-10 \log[S(2\omega_m, Q_0)] \approx 15.56 + 20 \log Q_0 \text{ dB}, \quad (17a)$$

$$-10 \log[S(3\omega_m, Q_0)] \approx 27.60 + 20 \log Q_0 \text{ dB}. \quad (17b)$$

Hence, the third harmonic rejection is about 12 dB higher than the second harmonic rejection. Based on the desired harmonic rejection specification, a suitable value of Q_0 can be selected using Eq. (17). Of course, the condition of Eq. (9) must be fulfilled for T network implementation.

4. Complex impedance matching

When the impedance to be matched is complex, the reactance in the termination impedance can be absorbed into the impedance matching network. Consider the circuit in Fig. 6 where the load impedance can be represented by a series combination of resistance R_2 and inductance L_{22} . We can define $L_2 = L_{22} + L_{21}$. The value of L_2 can be calculated using Eq. (11b). Hence, the required value of L_{21} can be easily determined.

In other words, the complex impedance matching problem can be converted into a pure resistance matching

with termination of R_2 . If the source impedance is complex, it can be similarly considered as a combination of source resistance R_1 and a source reactance. A doubly complex impedance terminated network can be treated as a pure resistance matching problem with source resistance R_1 and load resistance R_2 . The source reactance and load reactance can be absorbed into the impedance matching T network.

The T network design algorithm can be summarized as follows:

1. Convert the source and load impedances into series combinations of resistance and reactance.
2. Determine the required harmonic rejection and calculate the require Q_0 using Fig. 5 or Eq. (17).
3. Based on the selected Q_0 , determine Q_1 and Q_2 from Eq. (10).
4. Calculate the T network inductances and capacitance using Eq. (11).
5. Absorb the source and load reactances into the T network.

From the point of view of the dual theorem [18], both pi and T matching networks have the same theoretical performance. A circuit designer may choose either pi or T network solutions based on the complex impedance to be matched. When the impedance is inductive, T network design is preferable for the effective application of the absorption method described above to deal with the termination reactance. With the equations given in this paper, the desired harmonic rejection can be realized in a systematic manner.

5. Conclusion

A Q-based T network impedance matching method is developed. A practical guide is provided for selection of loaded Q based on the desired harmonic rejection specification. Using the design equations, the required inductance and capacitance of the T network can then be determined. The limits of the network are analytically demonstrated. It is shown that the third harmonic rejection is about 12 dB lower than second harmonic rejection provided that the loaded Q is larger than twice the minimum Q.

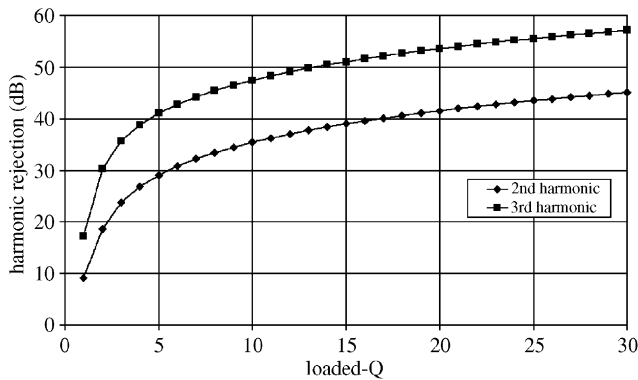


Fig. 5. Harmonic rejection performance of T impedance matching network.

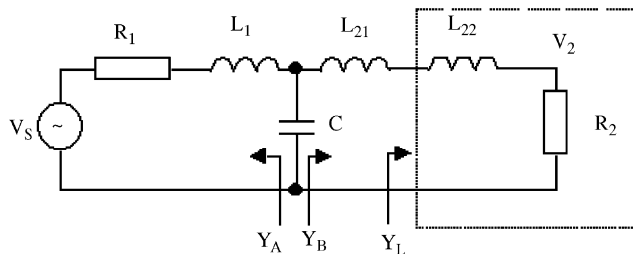


Fig. 6. T network impedance matching of complex load.

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