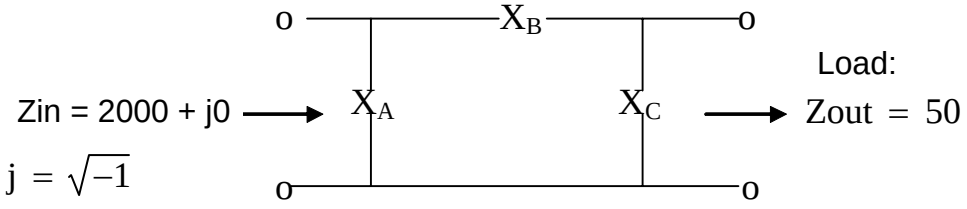
Pi-Network Analysis

CLC Pi-network Analysis with Lossless Elements

To match between impedances Z = 2000 ohms and Z = 50 ohms

Forward Direction, network terminated in 50 ohms

Tuning 130° pi-network to resonance. Zin = 2000 ohms, Zload = 50 ohms. Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$j = \sqrt{-1}$

$Xa = -j \cdot 219.89571936$ $Xb = j \cdot 242.24452283$ $Xc = -j \cdot 47.82388655$

$Za = Xa$ $Zb = Xb$ $Zc = Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Zc}}$$

$Za = -219.89571936j$

$Zb = 242.24452283j$

$Zc = -47.82388655j$

$Z1 = 23.88828935 - 24.97526976j$

$Z2 = Z1 + Zb$

$Z2 = 23.88828935 + 217.26925307j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Za}}$$

$Zin = 2 \times 10^3$

With lossless elements Zin is a purely resistive 2000 + j0 ohms when the network is resonant.

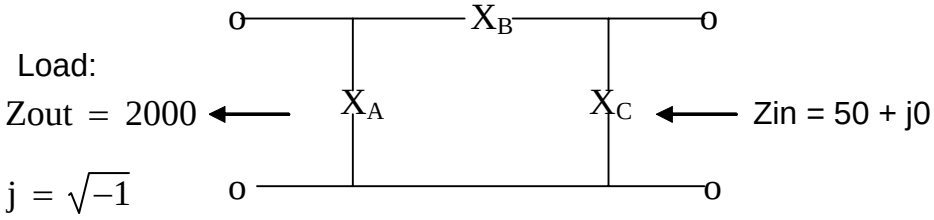
Fig 24-1

CLC Pi-network Analysis with Lossless Elements

To match between impedances Z = 2000 ohms and Z = 50 ohms

Reverse Direction, network terminated in 2000 ohms

Tuning 130° pi-network to resonance. Zin = 2000 ohms, Zload = 50 ohms. Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$Xa = -j \cdot 219.89571936$ $Xb = j \cdot 242.24452283$ $Xc = -j \cdot 47.82388655$

$Za = Xa$ $Zb = Xb$ $Zc = Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Za}}$$

$Za = -219.89571936j$

$Zb = 242.24452283j$

$Zc = -47.82388655j$

$Z1 = 23.88828935 - 217.26925307j$

$Z2 = Z1 + Zb$

$Z2 = 23.88828935 + 24.97526976j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Zc}}$$

$Zin = 50$ With lossless elements Zin is a purely resistive 50 + j0 ohms when the network is resonant.

Fig 24-2

CLC Pi-network Analysis with Real Elements, Untuned

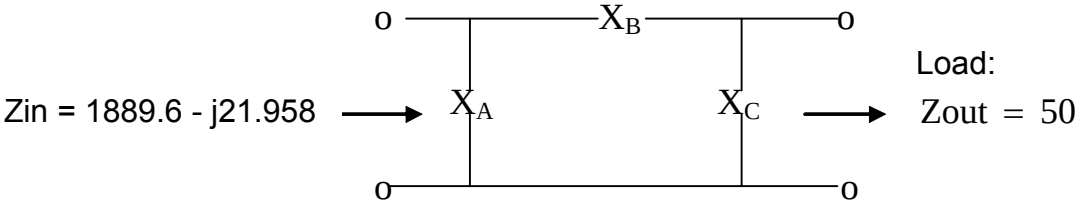
To match between impedances Z = 2000 ohms and Z = 50 ohms

Capacitor Q = 1000, Inductor Q = 200

Forward Direction, Network Terminated in 50 ohms

Reactance values <u>unchanged</u> from those during calculations with lossless elements

Tuning 130° pi-network to resonance. Zin = 2000 ohms, Zload = 50 ohms. Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$Qc = 1000$ $QL = 200$ $j = \sqrt{-1}$

$Xa = -j \cdot 219.89571936$ $Xb = j \cdot 242.24452283$ $Xc = -j \cdot 47.82388655$

$$Ra = \frac{-Xa}{j \cdot Qc} \qquad Rb = \frac{Xb}{j \cdot QL} \qquad Rc = \frac{-Xc}{j \cdot Qc}$$

$Za = Ra + Xa$ $Zb = Rb + Xb$ $Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Zc}}$$

Element Loss Resistances

$Za = 0.2199 - 219.89572j$ $Ra = 0.2199$

$Zb = 1.21122 + 242.24452j$ $Rb = 1.21122$

$Z1 = 23.88941 - 24.95033j$ $Zc = 0.04782 - 47.82389j$ $Rc = 0.04782$

$Z2 = Z1 + Zb$

$Z2 = 25.10063 + 217.29419j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Za}}$$

$Zin = 1.88956 \times 10^3 - 21.95809j$

Note that with lossy elements, and with the reactance values of the original lossless elements left unchanged, the resistive input impedance is now somewhat less than 2000 ohms, and academically, a significant amount of capacitive reactance has been introduced by the resistive loss in the elements, indicating that a perfect conjugate match no longer exists. However, from a practical viewpoint, the conjugate mismatch is so small as to be negligible with respect to loss in delivery of power to the load, and can be ignored. Resistive loss due to Rb = 0.2228 dB.

Conjugate Mismatch SWR	Reflection Coefficient RHO (ρ)	Conjugate Power Delivered	Conjugate Mismatch Loss
1.0594	0.0288	0.9992	0.0036 dB

Fig 24-3

CLC Pi-network Analysis with Real Elements, Untuned

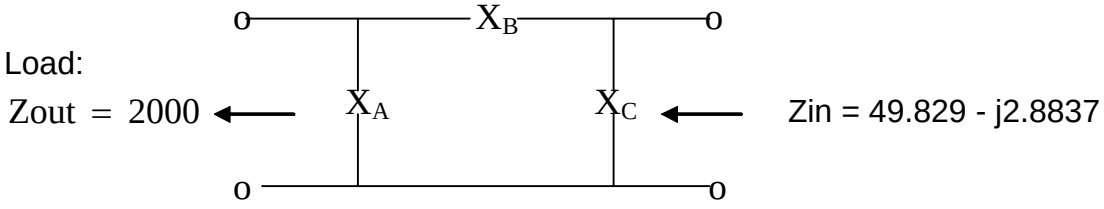
To match between impedances Z = 2000 ohms and Z = 50 ohms

Capacitor Q = 1000, Inductor Q = 200

Reverse Direction, Network Terminated in 2000 ohms

Reactance values <u>unchanged</u> from those during calculations with lossless elements

Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$QC = 1000$ $\quad QL = 200$ $\quad j = \sqrt{-1}$

$Xc = -j \cdot 219.89572$ $\quad Xb = j \cdot 242.244523$ $\quad Xa = -j \cdot 47.823887$

$$Ra = \frac{-Xa}{j \cdot QC} \qquad Rb = \frac{Xb}{j \cdot QL} \qquad Rc = \frac{-Xc}{j \cdot QC}$$

$Za = Ra + Xa$ $\quad Zb = Rb + Xb$ $\quad Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Zc}}$$

$Z1 = 24.10035 - 217.22206j$

$Z2 = Z1 + Zb$

$Z2 = 25.31157 + 25.02247j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Za}}$$

$Zin = 49.82903 - 2.88367j$

Element Loss Resistances

$Za = 0.04782 - 47.82389j$ $\quad Ra = 0.04782$

$Zb = 1.21122 + 242.24452j$ $\quad Rb = 1.21122$

$Zc = 0.2199 - 219.89572j$ $\quad Rc = 0.2199$

Note that with lossy elements, and with the reactance values of the original lossless elements left unchanged, the resistive input impedance is now somewhat less than 50 ohms, and academically, a significant amount of capacitive reactance has been introduced by the resistive loss in the elements, indicating that a perfect conjugate match no longer exists. However, from a practical viewpoint, the conjugate mismatch is so small as to be negligible with respect to loss in delivery of power to the load, and can be ignored. Loss due to Rb = 0.2228 dB.

Conjugate Mismatch SWR	Reflection Coefficient RHO	Conjugate Power Delivered	Conjugate Mismatch Loss
1.0596	0.0289	0.9992	0.0036 dB

Fig 24-4

CLC Pi-network Analysis with Real Elements, Retuned

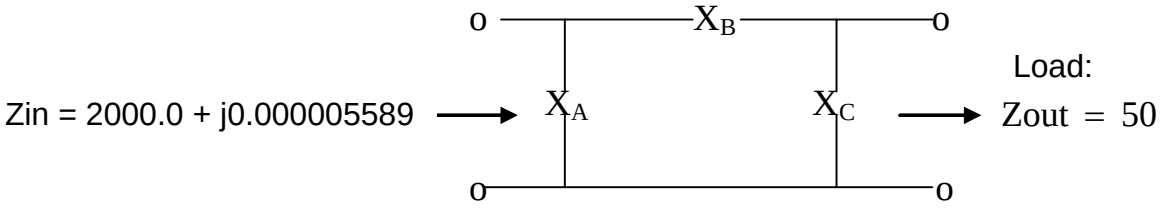
Capacitor Q = 1000, Inductor Q = 200

To match between impedances Z = 2000 ohms and Z = 50 ohms

Forward Direction, Network Terminated in 50 ohms

Reactances <u>retuned</u> to resonance after inserting real elements

Tuning 130° pi-network to resonance. Zin = 2000 ohms, Zload = 50 ohms. Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$QC = 1000$ $\quad QL = 200$ $\quad j = \sqrt{-1}$

$Xa = -j \cdot 219.9787443$ $\quad Xb = j \cdot 242.24453156$ $\quad Xc = -j \cdot 45.1905$

$$Ra = \frac{-Xa}{j \cdot QC} \qquad Rb = \frac{Xb}{j \cdot QL} \qquad Rc = \frac{-Xc}{j \cdot QC}$$

$Za = Ra + Xa$ $\quad Zb = Rb + Xb$ $\quad Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Zc}}$$

$Z1 = 22.4827 - 24.84795j$

$Z2 = Z1 + Zb$

$Z2 = 23.69392 + 217.39658j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Za}}$$

$Zin = 2 \times 10^{3} + 5.58927j \times 10^{-6}$

Element Loss Resistances

$Za = 0.21998 - 219.97874j$ $\quad Ra = 0.21998$

$Zb = 1.21122 + 242.24453j$ $\quad Rb = 1.21122$

$Zc = 0.04519 - 45.1905j$ $\quad Rc = 0.04519$

Note that readjustments to the reactance values of each element from those of the lossless case compensate for the reactances introduced by the loss resistance appearing in the real elements, returning (retuning) the network to resonance. As a result, a conjugate match has been re-established in the forward direction. Resistive loss due to Rb = 0.2228 dB.

.

Fig 24-5

CLC Pi-network Analysis with Real Elements

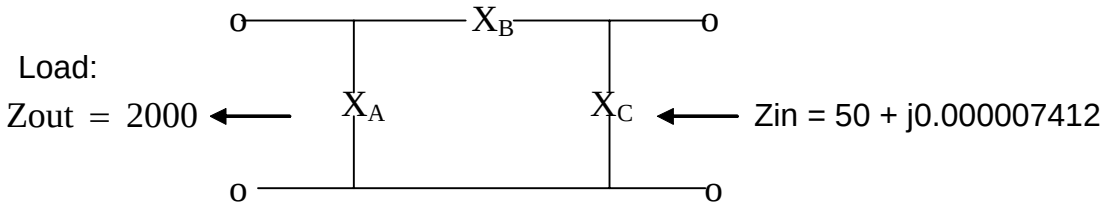
Capacitor Q = 1000, Inductor Q = 200

To match between impedances Z = 2000 ohms and Z = 50 ohms

Reverse Direction, Network Terminated in 2000 ohms

Reactances <u>retuned</u> to resonance after inserting real elements

Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$QC = 1000$ $\quad QL = 200$ $\quad j = \sqrt{-1}$

$Xa = -j \cdot 228.6958996$ $\quad Xb = j \cdot 250.613483$ $\quad Xc = -j \cdot 54.7973$

$$Ra = \frac{-Xa}{j \cdot QC} \qquad Rb = \frac{Xb}{j \cdot QL} \qquad Rc = \frac{-Xc}{j \cdot QC}$$

$Za = Ra + Xa$ $\quad Zb = Rb + Xb$ $\quad Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Za}}$$

$Z1 = 26.03328 - 225.69324j$

$Z2 = Z1 + Zb$

$Z2 = 27.28635 + 24.92024j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Zc}}$$

$Zin = 50 + 7.4119j \times 10^{-6}$

$Za = 0.2287 - 228.6959j$

$Zb = 1.25307 + 250.61348j$

$Zc = 0.0548 - 54.7973j$

Element Loss Resistances

$Ra = 0.2287$

$Rb = 1.25307$

$Rc = 0.0548$

Note that readjustments to the reactance values of each element from those of the lossless case compensate for the reactances introduced by the loss resistance appearing in the real elements, returning (retuning) the network to resonance. As a result, a conjugate match has been re-established in the reverse direction. Loss due to Rb = 0.2228 dB.

Fig 24-6

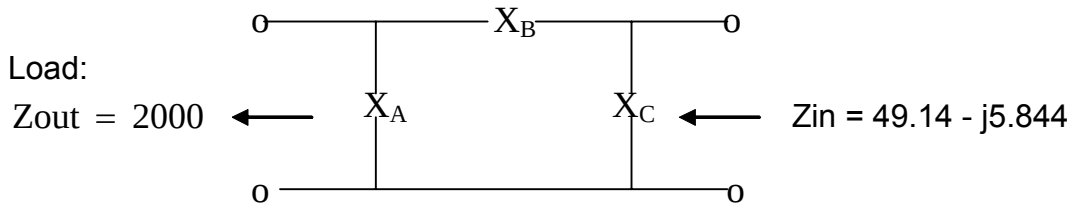
CLC Pi-network Analysis with Real Elements

To match between impedances Z = 2000 ohms and Z = 50 ohms

Capacitor Q = 1000, Inductor Q = 200

Tuned in Forward Direction, Network terminated in 50 ohms, then ReCalculated in <u>Reverse</u> Direction Terminated in 2000 ohms

Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms.



$QC = 1000$ $\quad QL = 200$ $\quad j = \sqrt{-1}$

$Xa = -j\cdot 219.979330045$ $\quad Xb = j\cdot 242.244503$ $\quad Xc = -j\cdot 45.2$

$$Ra = \frac{-Xa}{j\cdot QC} \qquad Rb = \frac{Xb}{j\cdot QL} \qquad Rc = \frac{-Xc}{j\cdot QC}$$

$Za = Ra + Xa$ $\quad Zb = Rb + Xb$ $\quad Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Za}}$$

$Z1 = 24.11837 - 217.30266j$

$Z2 = Z1 + Zb$

$Z2 = 25.3296 + 24.94185j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Zc}}$$

$Zin = 49.13959 - 5.8441j$

Element Loss Resistances

$Za = 0.21998 - 219.97933j$	$Ra = 0.21998$
$Zb = 1.21122 + 242.2445j$	$Rb = 1.21122$
$Zc = 0.0452 - 45.2j$	$Rc = 0.0452$

Network first tuned to resonance in forward direction, and then calculated to show amount of conjugate mismatch when going in reverse direction without retuning. Mismatch loss increased to 0.0154 dB from 0.0036 dB when going in reverse direction with reactance values of lossy elements unchanged from being tuned to resonance in forward direction. Total loss, combination of mismatch and resistive losses = 0.2382 dB

Conjugate Mismatch SWR	Reflection Coefficient RHO (ρ)	Conjugate Power Delivered	Conjugate Mismatch Loss
1.1265	0.059	0.9965	0.0154 dB

Fig 24-7

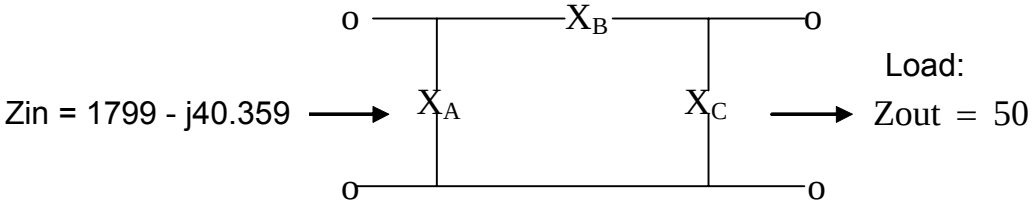
CLC Pi-network Analysis with Real Elements

Capacitor Q = 1000, Inductor Q = 200

To match between impedances Z = 2000 ohms and Z = 50 ohms

Tuned in Reverse Direction, Network Terminated in 2000 ohms, then ReCalculated in <u>Forward</u> Direction Terminated in 50 ohms.

Initial network parameters at resonance with lossless elements: Xa = - 219.89571936 ohms, Xb = 242.24452283 ohms, Xc = - 47.82388655 ohms ohms. .



$QC = 1000$ $\quad QL = 200$ $\quad j = \sqrt{-1}$

$Xa = -j \cdot 228.6959$ $\quad Xb = j \cdot 250.61345$ $\quad Xc = -j \cdot 54.7973$

$$Ra = \frac{-Xa}{j \cdot QC} \qquad Rb = \frac{Xb}{j \cdot QL} \qquad Rc = \frac{-Xc}{j \cdot QC}$$

$Za = Ra + Xa$ $\quad Zb = Rb + Xb$ $\quad Zc = Rc + Xc$

$$Z1 = \frac{1}{\frac{1}{Zout} + \frac{1}{Zc}}$$

$Z1 = 27.2818 - 24.87066j$

$Z2 = Z1 + Zb$

$Z2 = 28.53487 + 225.74279j$

$$Zin = \frac{1}{\frac{1}{Z2} + \frac{1}{Za}}$$

$Zin = 1.79923 \times 10^3 - 40.35882j$

$Za = 0.2287 - 228.6959j$

$Zb = 1.25307 + 250.61345j$

$Zc = 0.0548 - 54.7973j$

Element Loss Resistances

$Ra = 0.2287$

$Rb = 1.25307$

$Rc = 0.0548$

Network first tuned to resonance in reverse direction, and then calculated to show amount of conjugate mismatch when going in fprward direction without retuning. Mismatch loss increased to 0.0127 dB from 0.0036 dB when going in reverse direction with reactance values of lossy elements unchanged from being tuned to resonance in forward direction. Loss due to resistive loss Rb = 0.2228 dB. Total loss, combination of mismatch and resistive losses = 0.2355 dB

Conjugate Mismatch SWR	Reflection Coefficient RHO (ρ)	Conjugate Power Delivered	Conjugate Mismatch Loss
1.1141	0.0540	0.9971	0.0127 dB

Fig 24-8

W2DU 01022006

Comparison of Lossless vs Real Elements and Forward vs Reverse Directions in Pi-Networks

(CLC) Capacitor Q = 1000, Inductor Q = 200, Phase Delay 130 Degrees

Pi-network with Lossless Elements

Forward Direction, 50-Ohm Load

Zin = 2000 + j0.0000357787 (Fig 24-1)

Pi-network with Real Elements, Untuned

Forward Direction, 50-Ohm Load

Reactance values <u>unchanged</u> (Fig 24-3)

Zin = 1889.6 - j21.958

SWR = 1.0594, 0.0036 dB Conjugate Mismatch Loss

Pi-network, Real Elements, Retuned

Forward Direction, 50-Ohm Load

Reactances <u>retuned</u> to resonance (Fig 24-5)

Zin = 2000.0 + j0.000005589

Pi-network, Real Elements

Tuned in Reverse Direction with 2000-ohm Load, then ReCalculated in <u>Forward</u> Direction with 50-ohm Load

Zin = 1799 - j40.359 (Fig 24-7)

SWR = 1.1141, 0.0127 dB Conjugate Mismatch Loss

Fig 24-9

Pi-network with Lossless Elements

Reverse Direction, 2000-ohm Load

Zin = 50 + j0.0000002428 (Fig 24-2)

Pi-network with Real Elements, Untuned

Reverse Direction, 2000-ohm Load

Reactance values <u>unchanged</u> (Fig 24-4)

Zin = 49.829 - j2.8837

SWR = 1.0596, 0.0036 dB Conjugate Mismatch Loss

Pi-network, Real Elements, Retuned

Reverse Direction, 2000-ohm Load

Reactances <u>retuned</u> to resonance (Fig 24-6)

Zin = 50 + j0.000002021

Pi-network, Real Elements

Tuned in Forward Direction with 50-ohm Load, then ReCalculated in <u>Reverse</u> Direction with 2000-ohm Load

Zin = 49.14 - j5.844 (Fig 24-8)

SWR = 1.1265, 0.0154 dB Conjugate Mismatch Loss

W2DU 01022006